Lesson 7

RC, RL, RESONANCE AND FILTERS

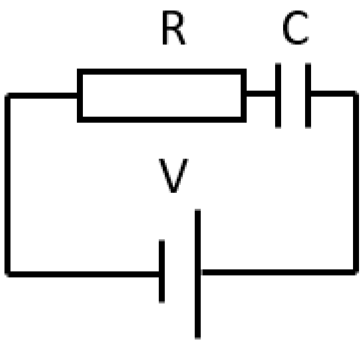
ACMA Syllabus February 2024 Chapter 3.2

Contents

Resistor Capacitor and Resistor Inductor Circuits

Series RC Circuit

If a resistor and capacitor are connected in series, the capacitor charges gradually through the resistor until the voltage across the resistor is equal to the supply voltage.



Series RC circuit

Figure 1

The time, in seconds, for the capacitor to charge depends on the resistor in Ohms and capacitor in Farads.

$$T = C \; x \; R$$

Example: 1 MΩ resistor, 6 uF capacitor with a V of 12 V,

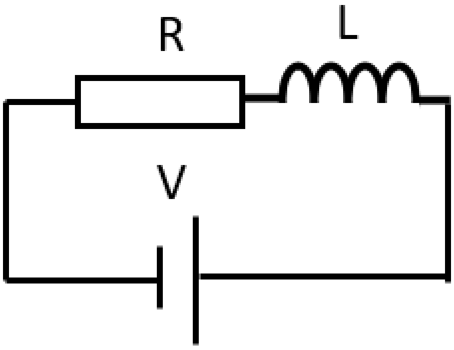
$$T = C \; x \; R$$

$$= 0.000006 \; x \; 1{,}000{,}000$$

= 6 Seconds

Series RL Circuit

If a resistor and inductor are connected in series, the self-inductance of the coil will resist the current flow as the magnetic field is generated. Once the field reaches its peak only the resistor will limit current flow. The time for the field to generate can be calculated.



Series RL circuit

Figure 2

Time in seconds, resistance in Ohms and inductance in Henries.

$$T = \frac{L}{R}$$

Example: 1kΩ resistor, 14 mH inductor and V of 12 V.

$$T = \frac{L}{R}$$

$$= \frac{0.014}{1000}$$

= 14 seconds.

Resonance

Parallel LC Circuit

The useful thing about LC circuits is that with any combination of the C value and the L value, they will have a resonant frequency. At this resonant frequency, the circuits act differently than the components individually.

A parallel LC circuit, Figure 3, will block any signal that is operating on the resonant frequency. In a parallel LC circuit at resonance, the time taken for the capacitor to charge and discharge, is the same time taken for the inductor to charge and discharge.

The energy in the circuit will charge the capacitor and, as the capacitor discharges, the energy will charge the inductor. As the inductor discharges, the energy recharges the capacitor. So, at the resonant frequency, the parallel LC circuit will oscillate.

The combined reactance is high increasing the impedance. So, at resonance, the parallel tuned circuit has a high impedance to the alternating signal.

A signal at any other frequency will pass through the parallel LC circuit.

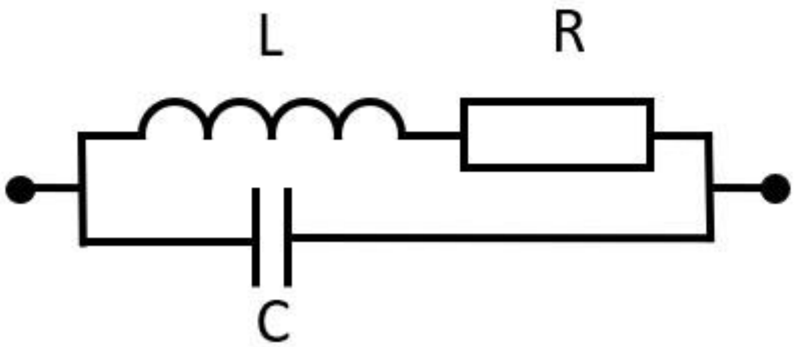


Figure 3 Parallel Circuit

- Low impedance below the resonant frequency (Path through the inductor)
- High Impedance at the resonant frequency (Signal is blocked)
- Low impedance above the resonant frequency (Path through the capacitor)

Series LC Circuit

In a series LC circuit, Figure 4, the energy goes through one component then the next. Any signal below the resonant frequency will be blocked by the capacitor. Signals above the resonant frequency will be blocked by the inductor. Signals at the resonant frequency are not blocked by the capacitor or the inductor.

At resonance in the series tuned circuit, the reactance cancels each other leaving only resistance in the circuit. So, at resonance, this circuit has a low impedance to the alternating signal.

Figure 4 Series Circuit

These LC circuits are an essential building block in communications equipment. They can act as filters, tuning circuit for a radio, or the basis of an oscillator.

- High impedance below the resonant frequency (Blocked by the capacitor)
- Low Impedance at the resonant frequency (Signal passes through the circuit)
- High impedance above the resonant frequency (Blocked by the inductor)

Impedance and Reactance

Every electronic component has a DC resistance. Even a length of wire has resistance, however small. The resistance to AC signals is called reactance. Reactance resists current without dissipating power, unlike resistors.

Inductive reactance increases with frequency and inductance.

Capacitive reactance decreases with frequency and capacitance.

Impedance is the combination of resistance and reactance and represents total opposition.

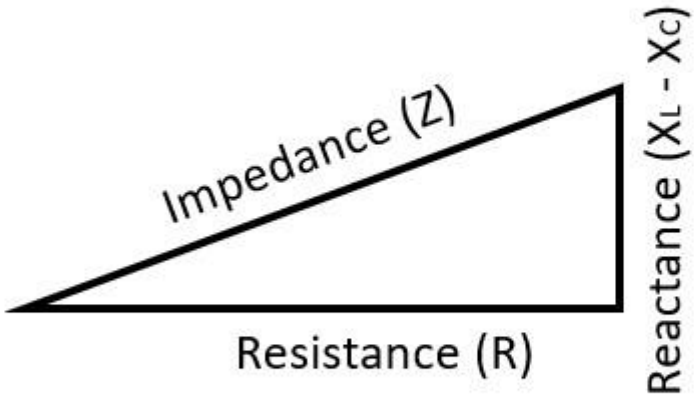


Figure 5

The reactance of a capacitor or inductor can be calculated depending on the frequency.

Inductor reactance (XL)

An inductor passes low frequencies and presents a high reactance to high frequencies.

At DC, the inductor resistance is that of the wire only. As the frequency increases, the reactance of the inductor to the AC signal also increases.

- Low frequencies – Low inductor reactance

- High frequencies – High inductor reactance

$$X_L = 2 \times \pi \times f \times L$$

X_L = Inductor reactance in Ohms
f = frequency in hertz (Hz)
L = Inductance in Henries
π = 3.141

Example: What is the inductive reactance of a 1H inductor at 30 MHz. (Choke)

$X_L = 2 \times \pi \times f \times L$
$= 2 \times 3.141 \times 30{,}000{,}000 \times 1$
$= 188\ M\Omega$

Now reduce the frequency to 4 MHz and calculate the inductive reactance.

$X_L = 2 \times \pi \times f \times L$
$= 2 \times 3.141 \times 4{,}000{,}000 \times 1$
$= 25\ M\Omega$

Capacitive Reactance

A capacitor passes high frequencies and presents a high reactance to low frequencies.

At DC, the capacitor resistance is infinitely high. As the frequency increases, the reactance of the capacitor to the AC signal decreases.

- Low frequencies – High capacitive reactance
- High frequencies – Low capacitive reactance

$$X_C = \frac{1}{2\pi f C}$$

X_C = Capacitive reactance in Ohms
f = frequency in hertz (Hz)
C = capacitance in Farads
π = 3.141

Example: What is the capacitive reactance of 0.01 uF capacitor at 30 MHz. (A HF bypass capacitor)

$$X_C = \frac{1}{2\pi f C}$$
$$= \frac{1}{2\ x\ 3.141\ x\ 30{,}000{,}000\ x\ 0.00000001}$$

$= 0.5\ \Omega$

Now reduce the frequency to 4 MHz and calculate the capacitive reactance.

$$X_C = \frac{1}{2\pi f C}$$
$$= \frac{1}{2\ x\ 3.141\ x\ 4{,}000{,}000\ x\ 0.00000001}$$

$= 3.97\ \Omega$

Reactance at Resonance (IMPORTANT)

At the resonant frequency of a series or parallel LC circuit, XL = XC and the impedance (Z) is reduced to R.

In a series tuned circuit, at resonance, there is a is a low resistance path for the signal through the circuit.

In a parallel tuned circuit, at resonance, there is a is a high resistance path for the signal through the circuit. XL and XC are equal and charge discharge times for the inductor and capacitor are in sync and the circuit will oscillate.

Frequency Calculation

$$f = \frac{1}{2\pi\sqrt{L\ x\ C}}$$

f = frequency in hertz (Hz)
C = capacitance in Farads (F)
L = inductance in Henries (L)
π = 3.141

Example: What is the resonant frequency of a capacitor of 1 uF and an inductor of 1 uH

$$f = \frac{1}{2\ x\ \pi\ x\ \sqrt{L\ x\ C}}$$

$$= \frac{1}{2\ x\ 3.141\ x\ \sqrt{0.000001\ x\ 0.000001}}$$

$$= \frac{1}{6.282\ x\ 0.000001}$$

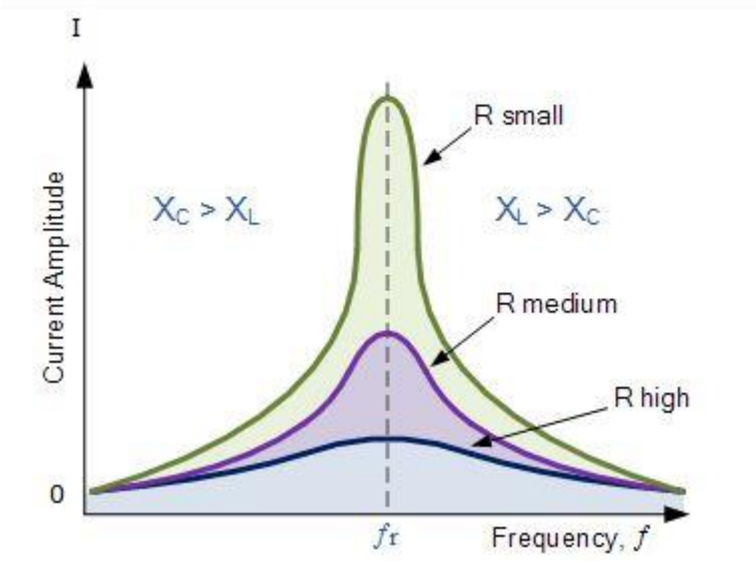
$$= \frac{1}{0.000006282}$$

= 159 kHz

Q Factor

The ratio of the reactance to the resistance, in an operational circuit, is called the Q (Quality Factor) of the circuit. If the current in the circuit is plotted against frequency, either side of the resonant frequency, the result will be a Q plot resembling a bell curve. The base of the bell and the height of the bell indicate the bandwidth of the circuit.

A sample Q plot of a series tuned circuit is shown in Figure 6. As the frequency drops, the capacitive reactance X_C is predominant while the inductive reactance X_L is predominant in the higher frequencies.



Series Circuit

$$Q = \frac{X}{RS} \text{ or } Q = \frac{2\Pi f L}{RS}$$

Q = Quality factor
X = Reactance of capacitor or inductor (Calculated from 2 π f L)
R_S = Series resistance in Ohms.

Example Series circuit XL and XC 350 Ω and a resistance of 10 Ω

$$Q = \frac{X}{RS}$$
$$= \frac{350}{10}$$
Q = 35

Parallel Circuit

Were R, L and C are in parallel, the lower the parallel resistance, the more effect it will have in damping the circuit and lower the Q. This is useful in filter design to determine the bandwidth.

$$Q = \frac{Rp}{X} \text{ or } Q = \frac{Rp}{2\ x\ \pi\ x\ f\ x\ L}$$

Q = Quality factor
X = Reactance of capacitor or inductor
R_P = Parallel resistance in Ohms.

Example Parallel circuit XL and XC 350 Ω and a resistance of 10 Ω

$$Q = \frac{\mathrm{RP}}{X}$$
$$= \frac{10}{350}$$
Q = 0.028

Bandwidth

The bandwidth (BW) is directly related to the frequency and Q of the circuit. The higher the Q, the narrower the bandwidth.

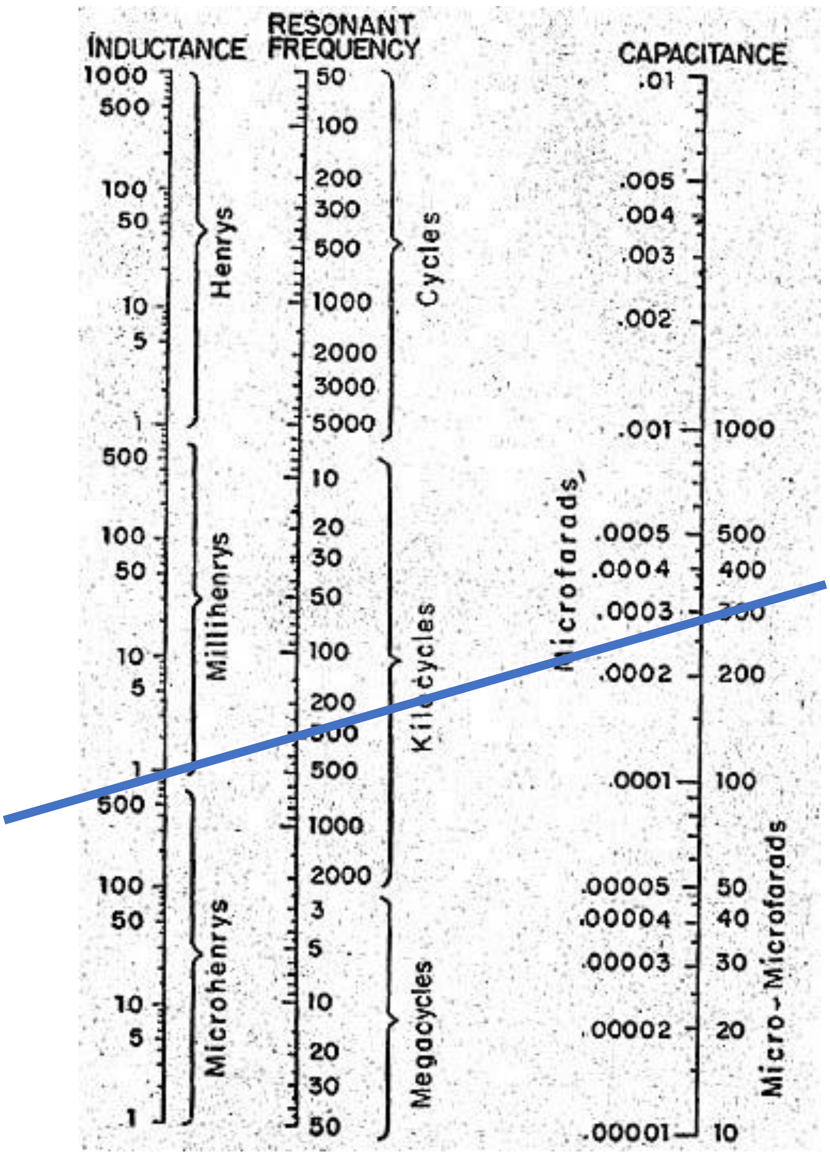
$$\text{BW} = \frac{f}{Q}$$

BW = hertz (Hz)
F = hertz (Hz)
Q = quality factor.

Example: What is the bandwidth of a 14 MHz circuit with a Q of 35?

BW = f / Q
= 14000000 /35
= 400,000 Hz

I found the following table in Coyne's Television and Radio Handbook P183. This gives a good starting point to match L and C for a particular frequency.
Draw a line from the desired capacitance to the desired inductance and the line will pass through the estimated resonant frequency. In the example shown, a 5 mH inductor and a 500 uF capacitor would resonate at approximately 100 kHz.



Filters

Electronic filters remove unwanted frequency components from the applied signal, enhance wanted ones or both.

Filters fall into four basic categories.

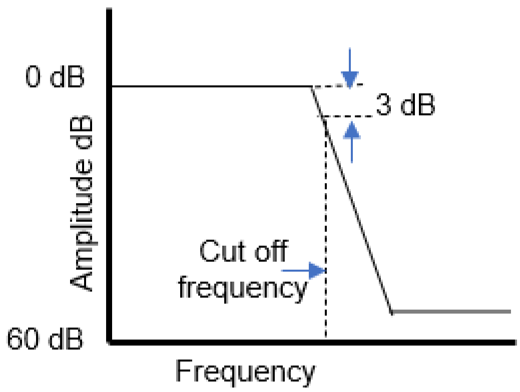
- Low pass (LPF)
- High pass (HPF)
- Band pass. (BPF)
- Band stop. (BSF)

Filters can be constructed with active and or passive components. Only passive components will be used in the following examples.

The cut off frequency is usually selected as the frequency where the filter loss is -3dB. The -3dB point is also known as the "half power" point.

Low-pass filters

A low-pass filter (LPF) only passes signals below its cutoff frequency while attenuating all signals above.



Low Pass Filter

Choke

A choke is a simple low pass filter. The inductor will pass signals until the Inductive reactance rises high enough to block the signals.

Example: What frequency would a 100mH choke reach 1 kΩ cut off?

$$X_L = 2 \times \pi \times f \times L$$

$$f = \frac{XL}{2\ x\ \pi\ x\ L}$$

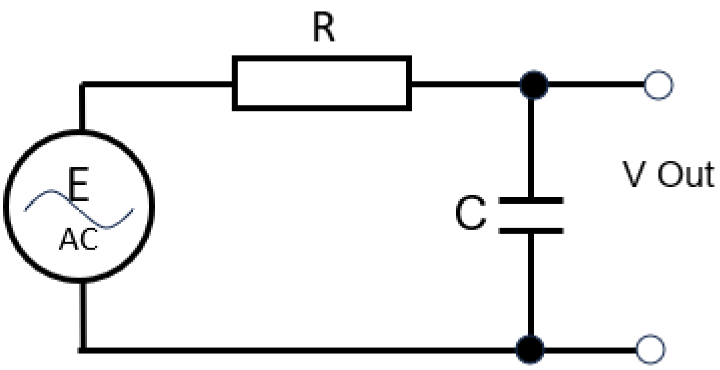
$$f = \frac{1000}{2\ x\ 3.141 x\ 0.1}$$

$$f = \frac{1000}{0.6282}$$

f = 1591.8 Hz

RC Filter

The RC low pass filter will allow low frequency signals to pass until the capactive reactance of C becomes low enough to pass signals.



Example: What size capacitor is needed to prevent signals above 1MHz passing to V Out XL = 10 Ohms)?

$$X_C = \frac{1}{2\pi f C}$$

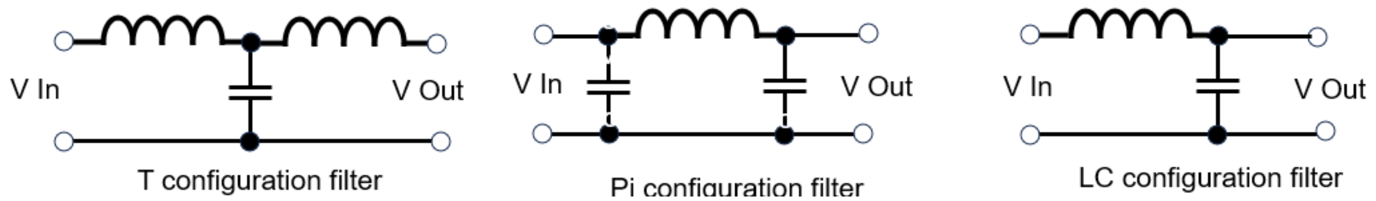
$$X_{C\ x\ 2\ x\ \pi\ x\ f} = \frac{1}{C}$$

$$10\ x\ 2\ x\ 3.141\ x\ 1000000 = \frac{1}{C}$$

$$62{,}820{,}000 = \frac{1}{C}$$

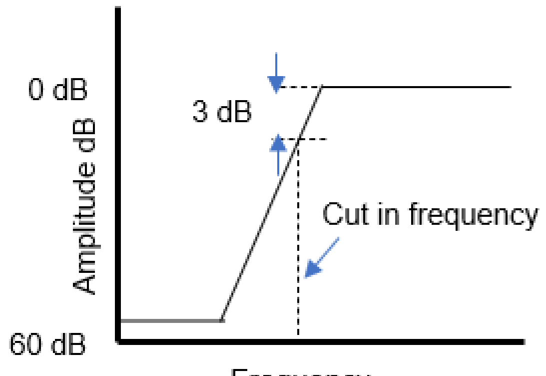
C= 15.9 nF

LC Low Pass Circuits



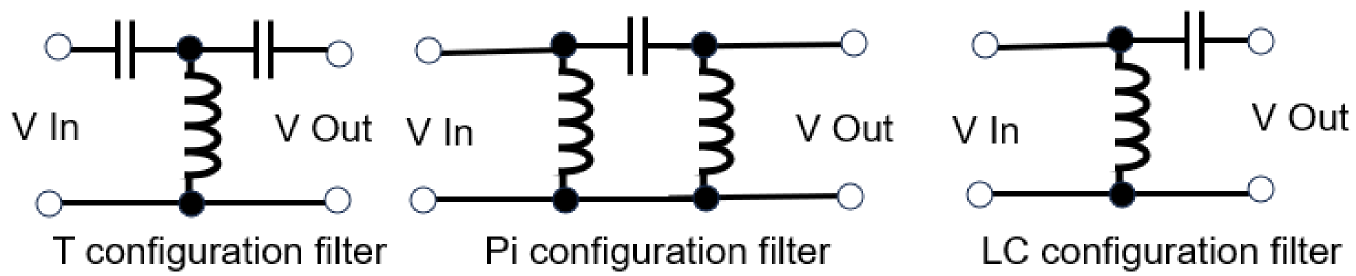
High-pass filters

A high-pass filter (HPF) only passes signals above its cutoff frequency and attenuates all signals below.



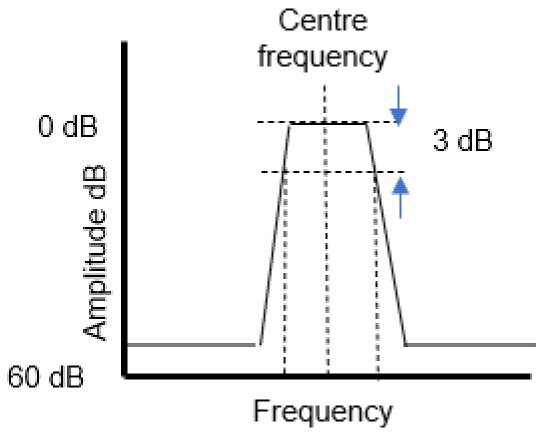
High Pass Filter

LC High Pass Circuits



Band-pass filters

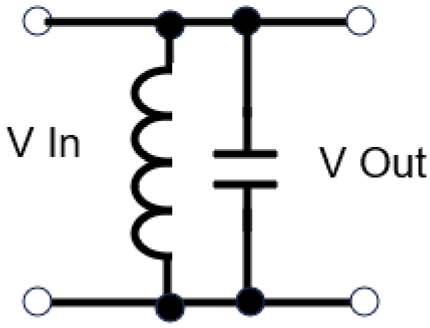
A band pass filter (BPF) allows signals within a selected range of frequencies to pass and any frequencies outside this band are attenuated.



Band Pass Filter

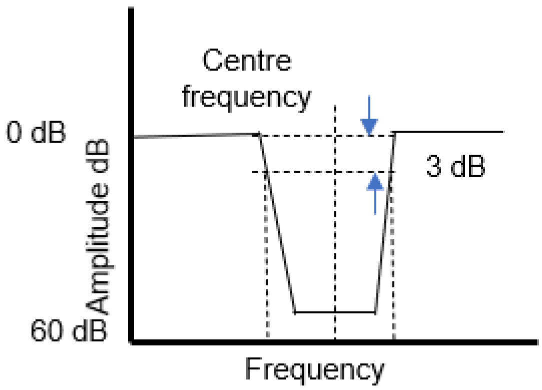
Configurations

The series LC circuit will only allow the resonant frequency to pass through and bloc other frequencies.
The parallel LC circuit will allow all frequencies, except the resonant frequencies to pass to earth.



Band-stop filters.

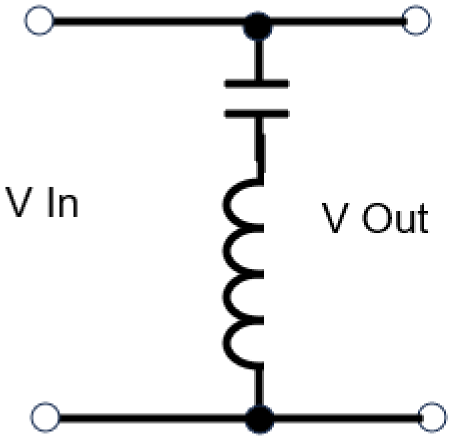
A band stop filter (BSF) attenuates signals within a selected range of frequencies and allows the frequencies outside this band to pass.

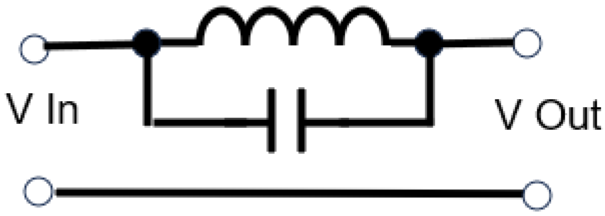


Band Stop Filter

Configurations

The series LC circuit will pass the resonant frequency to earth and by blocking other frequencies allow the signal to pass.
The parallel LC circuit will block the resonant frequency from passing.

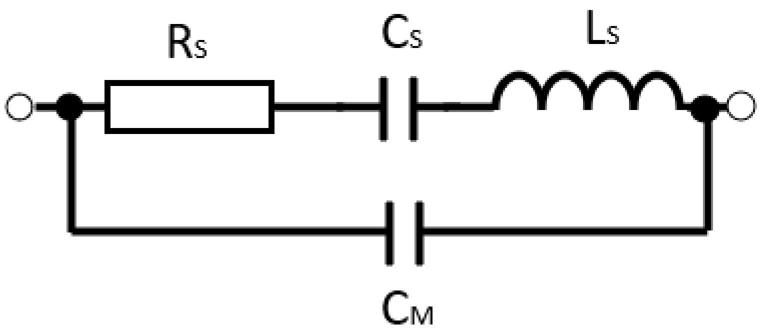




Quartz crystal Filters

A crystal is a piezoelectric device that uses mechanical resonance to achieve a very precise frequency output. The most common type of crystal material is quartz, but different piezoelectric materials may be used.

Piezo crystal



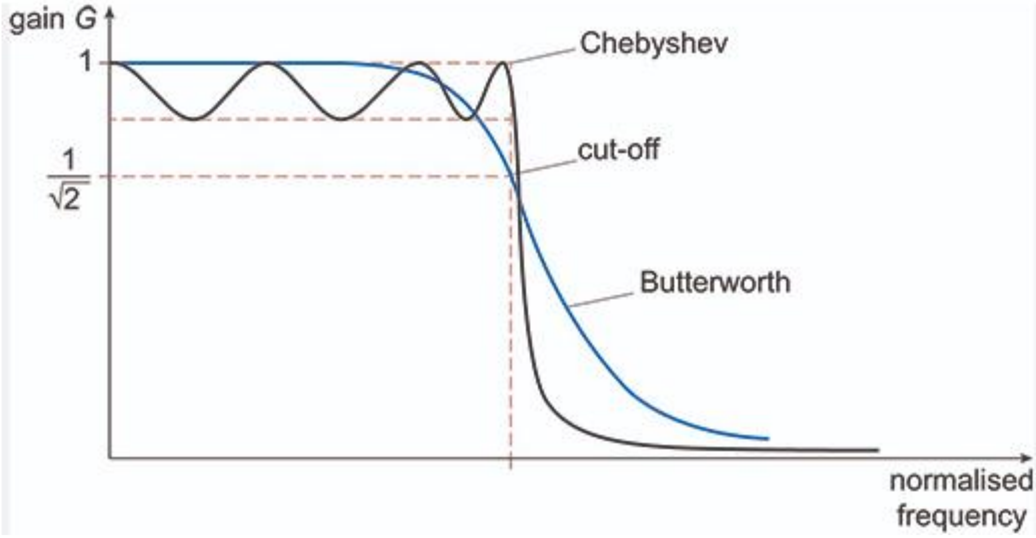
Crystal Equivalent Circuit

The above equivalent circuit consists of a series R_S-C_S-L_S circuit in parallel with a capacitance C_M. When the crystal mounted across the AC source is not vibrating, it is equivalent to the capacitance C_M. However, when the crystal vibrates, it acts like a tuned series R-L-C circuit.

Filter Variations

Butterworth Filter

The Butterworth filter is an analogue filter design which produces the best output response with no ripple in the pass band or the stop band resulting in a maximally flat filter response but at the expense of a wide transition band.



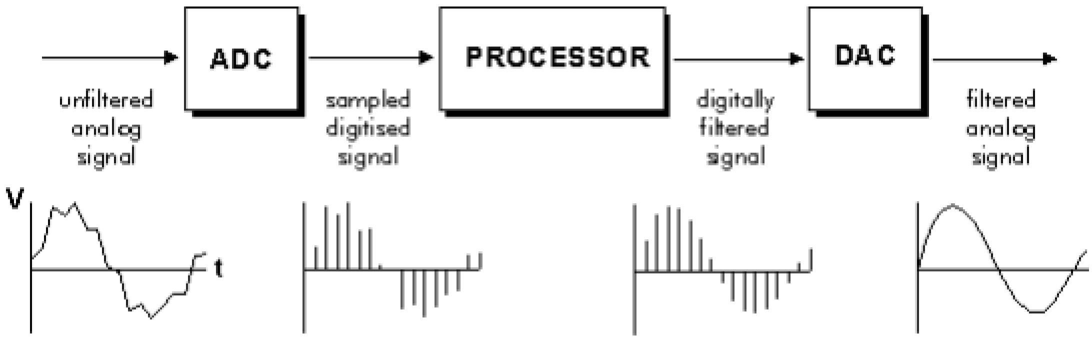
Chebyshev Filter

The Chebyshev filter is popular in RF application - using inductor and capacitor, LC combinations to provide a fast transition from passband to stopband. This transition comes at the cost of in-band ripple, and this may not make it suitable for all applications.

Digital Filters

As discussed previously, analog filter uses analog electronic circuits made from components such as resistors, capacitors and op amps to produce the required filtering effect. There are well-established standard techniques for designing an analog filter circuit for a given requirement.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialised DSP (Digital Signal Processor) chip.



Go to Lesson 7 questions.